EE-565 – W9
INDUCTION MACHINE – VECTOR CONTROL

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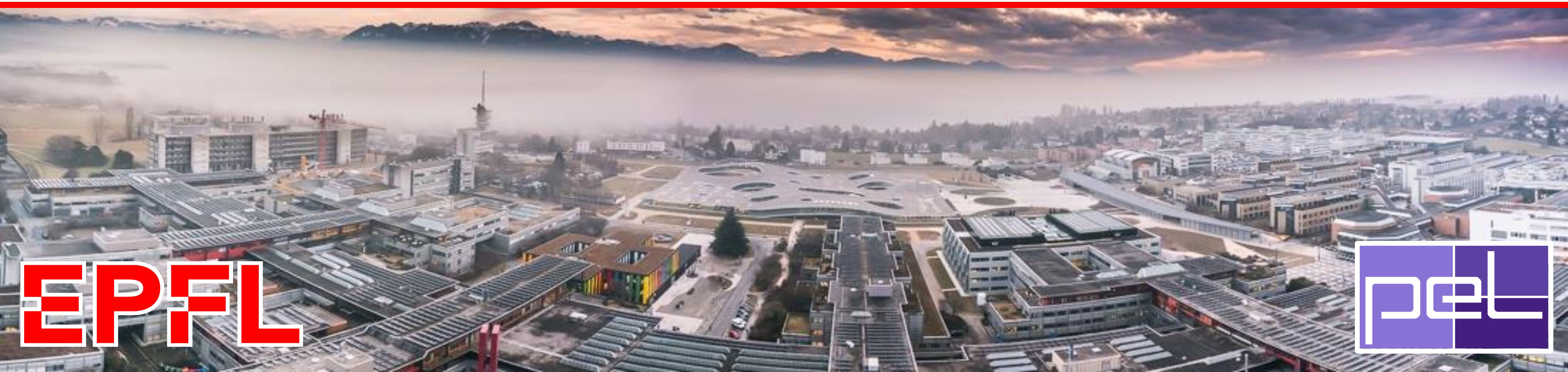


FLUX AND TORQUE DECOUPLING

Flux and Torque decoupling by modeling in the rotor flux oriented frame

INDUCTION MACHINE MODEL – MODEL IN A COMMON REFERENCE FRAME

Electrical Equations

$$\begin{cases} \underline{v}_s = R_s \cdot \underline{i}_s + \dfrac{\mathrm{d}\underline{\phi}_s}{\mathrm{dt}} + \mathrm{j}\omega_d \underline{\phi}_s \\ 0 = \underline{v}_r = R_r \cdot \underline{i}_r + \dfrac{\mathrm{d}\underline{\phi}_r}{\mathrm{dt}} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r \\ \underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \\ \underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \end{cases}$$

Mechanical Equations

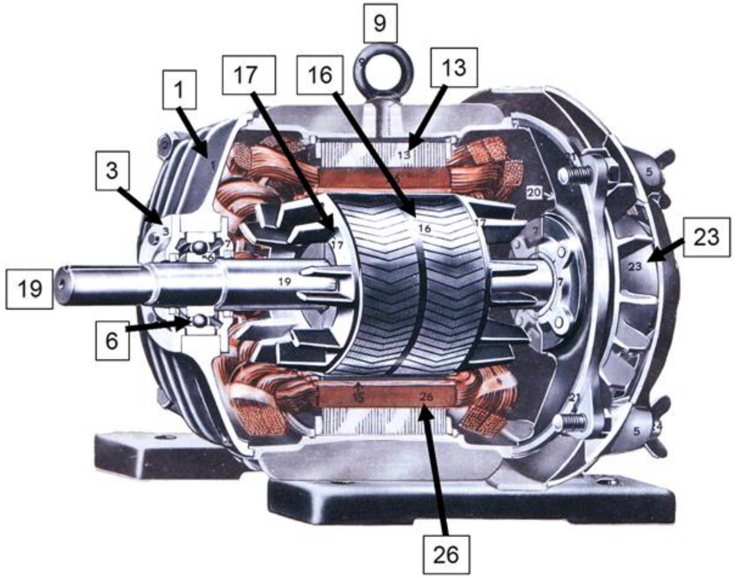
$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{dt}} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{dt}} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = \dfrac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{\underline{i}_s \cdot \hat{\underline{i}}_r\right\} \quad \text{(or equivalent)} \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m$$

$$\omega_e = P_p \omega_m$$

The subscripts <dq> have been removed to simplify the notation



VECTOR CONTROL BASICS

Instead of controlling magnitude and frequency of the stator currents, it is better to **control its components**

The control is formulated in a **rotating reference frame**, that is synchronized to the space vector of the flux (typically rotor flux)

Therefore, the vector control is also commonly named **Field Oriented Control (FOC)**

As shown in the following, this allows decoupling two actions:

- The **d-axis** component of the stator currents can control the **flux**
- The **q-axis** component of the stator currents can control the **torque**

This makes the control **similar to a DC-machine**, where:

- The **excitation current** can control the **flux**
- The **armature current** can control the **torque**

USEFUL PARAMETERS

Some additional parameters are introduced to simplify the expressions:

- **Coupling Factor**

$$k = \frac{L_m}{\sqrt{L_s\,L_r}}$$

- **Total Leakage Factor**

$$\sigma = 1 - k^2 = 1 - \frac{L_m^2}{L_s\,L_r}$$

- **Total Leakage Inductance**

$$L_\sigma = \sigma\,L_s = \frac{L_s\,L_r - L_m^2}{L_r}$$

With small leakage inductances $L_\sigma \approx L_{l,s} + L_{l,r}$

- **Stator and Rotor Time Constants**

$$T_s = \frac{L_s}{R_s} \qquad T_r = \frac{L_r}{R_r}$$

- **Stator and Rotor Transient Time Constants**

$$T_s' = \sigma\,T_s = \sigma\,\frac{L_s}{R_s} \qquad T_r' = \sigma\,T_r = \sigma\,\frac{L_r}{R_r}$$

INDUCTION MACHINE MODEL EXPRESSED IN TERMS OF ROTOR FLUXES

The fundamental variables in the induction machine model are fluxes and currents

Current and fluxes are linked to one another by linear relationship

It is possible to **select only two vector state variables** for the system (= 4 scalar variables)

Instead of expressing the equations in terms of the stator and rotor currents, it is more convenient to reformulate the model in terms of

- **Stator currents**
- **Rotor fluxes**

$$\begin{cases} \underline{v}_s = R_s \cdot \underline{i}_s + \dfrac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s \\ 0 = \underline{v}_r = R_r \cdot \underline{i}_r + \dfrac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r \\ \underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \\ \underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \\ T_{em} = \dfrac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s \cdot \hat{\underline{i}}_r \right\} \end{cases}$$

$$\begin{cases} k = \dfrac{L_m}{\sqrt{L_s \, L_r}} \qquad \sigma = 1 - k^2 = 1 - \dfrac{L_m^2}{L_s \, L_r} \\ L_\sigma = \sigma \, L_s = \dfrac{L_s \, L_r - L_m^2}{L_r} \\ T_s = \dfrac{L_s}{R_s} \qquad T_r = \dfrac{L_r}{R_r} \\ T_s' = \sigma \, T_s = \sigma \, \dfrac{L_s}{R_s} \qquad T_r' = \sigma \, T_r = \sigma \, \dfrac{L_r}{R_r} \end{cases}$$

INDUCTION MACHINE MODEL EXPRESSED IN TERMS OF ROTOR FLUXES

We need to **compute the stator fluxes and the rotor currents** as **functions of the stator currents and of the rotor fluxes**

We can use the relationship between the space vectors

$$\underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r$$

$$\underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s$$

From the second equation, we can compute the rotor currents as:

$$\underline{i}_r = \frac{1}{L_r}\underline{\phi}_r - \frac{L_m}{L_r}\underline{i}_s$$

By substituting in the first equation, the stator fluxes are expressed as:

$$\underline{\phi}_s = L_s \cdot \underline{i}_s + \frac{L_m}{L_r} \cdot \underline{\phi}_r - \frac{L_m^2}{L_r} \cdot \underline{i}_s = L_s\left(1 - \frac{L_m^2}{L_s L_r}\right) \cdot \underline{i}_s + \frac{L_m}{L_r} \cdot \underline{\phi}_r$$

$$= \sigma L_s \cdot \underline{i}_s + \frac{L_m}{L_r} \cdot \underline{\phi}_r = L_\sigma \cdot \underline{i}_s + \frac{L_m}{L_r} \cdot \underline{\phi}_r$$

ROTOR FLUX DYNAMICS

We can replace these expressions in the voltage balance equations

From the **rotor voltage balance equation,** it results:

$$0 = \underline{v}_r = R_r \cdot \underline{i}_r + \frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r = \frac{R_r}{L_r}\,\underline{\phi}_r - R_r\,\frac{L_m}{L_r}\,\underline{i}_s + \frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r$$

This expression can be rearranged as:

$$\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r + \frac{R_r}{L_r}\,\underline{\phi}_r = R_r\,\frac{L_m}{L_r}\,\underline{i}_s$$

By recalling the definition of the rotor time constant, it results

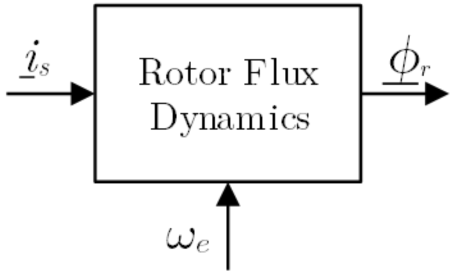
$$\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r + \frac{1}{T_r}\,\underline{\phi}_r = \frac{L_m}{T_r}\,\underline{i}_s$$

In this expression, **the stator current acts as the input of a dynamical system with the rotor flux as state variable**

We can control the rotor flux by controlling the stator currents

As could be expected, and as will be shown later, we can control the stator currents by controlling the stator voltages

This choice of variables is particularly **suited for implementing a cascaded control**



ROTOR FLUX DYNAMICS IN THE SYNCHRONOUS REFERENCE FRAME

The rotor flux is dynamically linked to the stator currents through the equation:

$$\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r + \frac{1}{T_r}\underline{\phi}_r = \frac{L_m}{T_r}\underline{i}_s$$

(valid for any arbitrary reference frame)

If **we choose our reference frame to be perfectly aligned with the space vector of the rotor fluxes**, then: $\theta_d = \angle\underline{\phi_r}$

$$\underline{\phi}_r = \phi_{r,d} = \phi_r$$

(the rotor flux is entirely on the d-axis)

By expressing the previous equation in the d-axis and in the q-axis components (i.e., real and imaginary part), it results:

$$\begin{cases} \dfrac{\mathrm{d}\phi_r}{\mathrm{d}t} + \dfrac{1}{T_r}\phi_r = \dfrac{L_m}{T_r} i_{s,d} \\ (\omega_d - \omega_e)\phi_r = \dfrac{L_m}{T_r} i_{s,q} \end{cases}$$

The **magnitude of the rotor flux** only depends on the **d-axis component of the stator current**

The **speed of the rotor flux** depends on the **rotor speed** and on the **q-axis component of the stator current**
It represents the **slip frequency of the rotor flux**

$$\omega_{slip} = \omega_d - \omega_e = \frac{L_m}{T_r}\frac{i_{s,q}}{\phi_r}$$

In steady state conditions, it is also equal to the slip frequency of the stator flux and to the frequency of the rotor currents

ELECTROMAGNETIC TORQUE IN THE SYNCHRONOUS REFERENCE FRAME

The expression of the electromagnetic torque in terms of stator currents and rotor fluxes is:

$$T_{em} = \frac{3}{2} P_p \frac{L_m}{L_r} \operatorname{Im}\left\{ \underline{i}_s \cdot \hat{\underline{\phi}}_r \right\}$$

(valid for any arbitrary reference frame)

If **we choose our reference frame to be perfectly aligned with the space vector of the rotor fluxes**, then: $\theta_d = \angle \underline{\phi_r}$

$$\underline{\phi}_r = \phi_{r,d} = \phi_r$$

(the rotor flux is entirely on the d-axis)

The rotor flux in the syncrhonous reference frame is a real quantity and can be taken out of the imaginary part

What is left is only the imaginary part of the stator currrent vector, which is the q-axis component

The torque is expressed as:

$$T_{em} = \frac{3}{2} P_p \frac{L_m}{L_r} \phi_r \, i_{s,q}$$

The **electromagnetic torque** only depends on the **q-axis component of the stator current**

FLUX AND TORQUE DECOUPLING

From the previous analysis

If **we choose our reference frame to be perfectly aligned with the space vector of the rotor fluxes**, then: $\theta_d = \angle \underline{\phi}_r$

$\underline{\phi}_r = \phi_{r,d} = \phi_r$ (the rotor flux is entirely on the d-axis)

We can conclude that:

- The **d-axis** component of the stator currents can control the **flux**
- The **q-axis** component of the stator currents can control the **torque**
- The **slip frequency** of the rotor flux depends on the **q-axis** component of the stator currents, and therefore on the required **torque**

$$\begin{cases} \dfrac{\mathrm{d}\phi_r}{\mathrm{d}t} + \dfrac{1}{T_r}\phi_r = \dfrac{L_m}{T_r} i_{s,d} \\ T_{em} = \dfrac{3}{2} P_p \dfrac{L_m}{L_r} \phi_r \, i_{s,q} \\ \omega_{slip} = \omega_d - \omega_e = \dfrac{L_m}{T_r} \dfrac{i_{s,q}}{\phi_r} \end{cases}$$

We induction machine behaves as an equivalent DC machine where:

- The **d-axis** component of the stator currents corresponds to the **excitation current**
- The **q-axis** component of the stator currents corresponds to the **armature current**

We can obtain a **Decoupled Control of Flux and Torque**

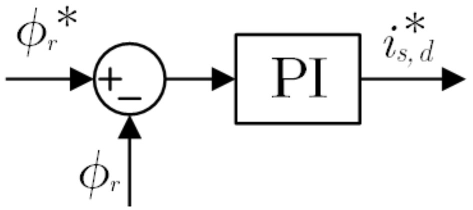
FLUX AND TORQUE CONTROL

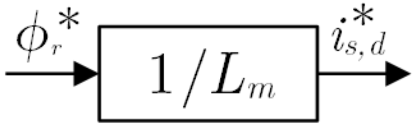
We can obtain a **Decoupled Control of Flux and Torque**

The dynamics of the rotor flux magnitude is determined by a **first order system**

- A **Proportional-Integral (PI)** controller can be used to track the desired flux
- In some cases, the flux is controlled in **open-loop**, and the d-axis current reference is simply set as

$$i_{s,d}^* = \phi_r^* / L_m$$ (from the steady-state relationship)

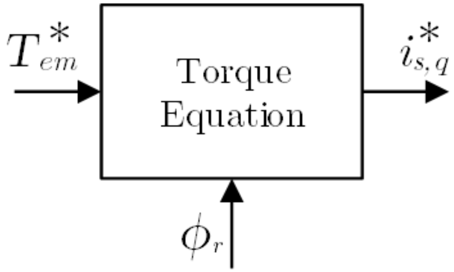




The electromagnetic torque is algebraically linked to the q-axis current

The reference q-axis current can be found from the reference torque as:

$$i_{s,q}^* = \frac{2}{3}\frac{L_r}{L_m}\frac{1}{P_p}\frac{1}{\phi_r}T_{em}^*$$



However, we need to examine two aspects:

- How do we control the stator currents? (our control inputs are the stator voltages)
- How do we obtain the information about the rotor flux? (phase and magnitude)

STATOR CURRENTS CONTROL

Control of the stator currents

STATOR CURRENT DYNAMICS

From the previous analysis, the stator fluxes have been expressed as functions of the stator currents and of the rotor fluxes as

$$\underline{\phi}_s = L_\sigma \cdot \underline{i}_s + \frac{L_m}{L_r} \cdot \underline{\phi}_r$$

This expression can be replaced in the stator voltage balance equation

$$\begin{aligned}\underline{v}_s &= R_s \cdot \underline{i}_s + \frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s \\ &= R_s \cdot \underline{i}_s + L_\sigma \frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \frac{L_m}{L_r}\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma \underline{i}_s + \mathrm{j}\omega_d \frac{L_m}{L_r}\underline{\phi}_r\end{aligned}$$

This expression includes the derivative of the rotor fluxes space vector

We can use the expression from the previously computed rotor flux dynamic equation

$$\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r + \frac{R_r}{L_r}\underline{\phi}_r = R_r \frac{L_m}{L_r}\underline{i}_s$$

And replace it in the voltage balance equation of the stator currents, to eliminate the derivative

STATOR CURRENT DYNAMICS

We can use the expression from the previously computed rotor flux dynamic equation

$$\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r + \frac{R_r}{L_r}\underline{\phi}_r = R_r\frac{L_m}{L_r}\underline{i}_s$$

This expression can be replaced in the stator voltage balance equation

$$\begin{aligned}
\underline{v}_s &= R_s \cdot \underline{i}_s + \frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} + \mathrm{j}\omega_d\underline{\phi}_s \\
&= R_s \cdot \underline{i}_s + L_\sigma\frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \frac{L_m}{L_r}\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma\underline{i}_s + \mathrm{j}\omega_d\frac{L_m}{L_r}\underline{\phi}_r \\
&= R_s \cdot \underline{i}_s + L_\sigma\frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma\underline{i}_s + \frac{L_m}{L_r}\left(\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}\omega_d\underline{\phi}_r\right) \\
&= R_s \cdot \underline{i}_s + L_\sigma\frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma\underline{i}_s + \frac{L_m}{L_r}\left(\mathrm{j}\omega_e\underline{\phi}_r - \frac{R_r}{L_r}\underline{\phi}_r + R_r\frac{L_m}{L_r}\underline{i}_s\right)
\end{aligned}$$

STATOR CURRENT DYNAMICS

We can isolate the terms that depend on the **stator currents** and the terms that depend on the **rotor flux**

$$\underline{v}_s = R_s \cdot \underline{i}_s + L_\sigma \frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma \underline{i}_s + \frac{L_m}{L_r}\left(\mathrm{j}\omega_e \underline{\phi}_r - \frac{R_r}{L_r}\underline{\phi}_r + R_r\frac{L_m}{L_r}\underline{i}_s\right)$$

$$= \left(R_s + R_r\frac{L_m^2}{L_r^2}\right)\cdot \underline{i}_s + L_\sigma \frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma \underline{i}_s + \frac{L_m}{L_r}\left(\mathrm{j}\omega_e \underline{\phi}_r - \frac{R_r}{L_r}\underline{\phi}_r\right)$$

$$= \underbrace{R_{tot}}_{\text{Equivalent Resistance}} \cdot \underline{i}_s + L_\sigma \frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma \underline{i}_s + \underbrace{\underline{e}_{eq}}_{\text{Equivalent back-EMF}}$$

The dynamics of the stator current is **equivalent to the dynamics of an RL circuit with a series-connected voltage source**

- **Equivalent Resistance**

$$R_{tot} = R_s + R_r\frac{L_m^2}{L_r^2}$$

- **Equivalent Inductance**

$$L_\sigma = \frac{L_s\, L_r - L_m^2}{L_r}$$

- **Equivalent back-EMF**

$$\underline{e}_{eq} = e_d + \mathrm{j}\, e_q = \left(\mathrm{j}\omega_e - \frac{R_r}{L_r}\right)\frac{L_m}{L_r}\,\underline{\phi}_r$$

STATOR CURRENT DYNAMICS

The dynamics of the stator current is **equivalent to the dynamics of an RL circuit with a series-connected voltage source**

$$\underline{v}_s = R_{tot} \cdot \underline{i}_s + L_\sigma \frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma \underline{i}_s + \underline{e}_{eq}$$

By expressing this equation in the synchronous refernece frame and by separating the d-axis and q-axis components, it results:

$$\begin{cases} v_{s,d} = R_{tot} \cdot i_{s,d} + L_\sigma \dfrac{\mathrm{d}i_{s,d}}{\mathrm{d}t} - \omega_d L_\sigma i_{s,q} - \dfrac{R_r L_m}{L_r^2}\phi_r \\ v_{s,q} = R_{tot} \cdot i_{s,q} + L_\sigma \dfrac{\mathrm{d}i_{s,q}}{\mathrm{d}t} + \omega_d L_\sigma i_{s,d} + \omega_e \dfrac{L_m}{L_r}\phi_r \end{cases}$$

It can be concluded that:

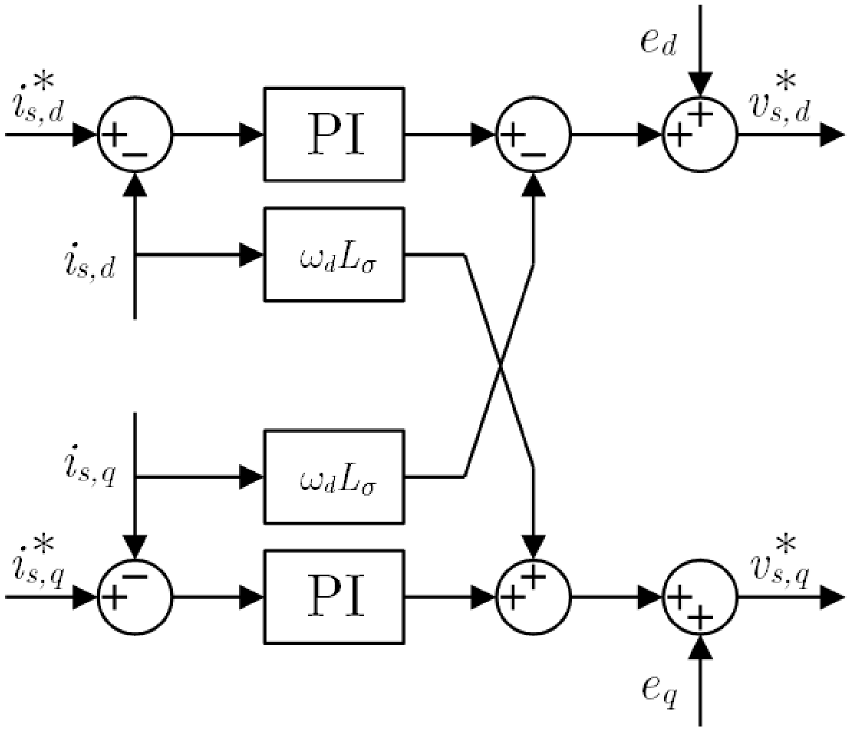
- The **d-axis** and **q-axis** components of the stator currents are **dynamically coupled** (need for cross-decoupling action)
- The **equivalent back-EMF** components depend on the **rotor flux** (need for feed-forward compensation)
- The **equivalent back-EMF in the q-axis component** depends on the **rotor speed**

STATOR CURRENT CONTROL

The stator current dynamics are the same of an RL-circuit with a series voltage source

The d-axis and q-axis components can be decoupled with a standard cross-decoupling algorithm (same as in grid-connected converters)

$$\begin{cases} v_{s,d} = R_{tot} \cdot i_{s,d} + L_\sigma \dfrac{\mathrm{d}i_{s,d}}{\mathrm{d}t} - \omega_d L_\sigma i_{s,q} - \dfrac{R_r L_m}{L_r^2} \phi_r \\ v_{s,q} = R_{tot} \cdot i_{s,q} + L_\sigma \dfrac{\mathrm{d}i_{s,q}}{\mathrm{d}t} + \omega_d L_\sigma i_{s,d} + \omega_e \dfrac{L_m}{L_r} \phi_r \end{cases}$$



The equivalent back-EMFs can be added in feed-forward as:

$$e_d = -\frac{R_r L_m}{L_r^2} \phi_r \qquad e_q = \omega_e \frac{L_m}{L_r} \phi_r$$

We need to examine one last aspect

- How do we obtain the information about the **rotor flux**? (phase and magnitude)

FIELD WEAKENING

Consider the stator equation, previously obtained in terms of stator currents and rotor fluxes

$$\underline{v}_s = R_s \cdot \underline{i}_s + L_\sigma \frac{\mathrm{d}\underline{i}_s}{\mathrm{d}t} + \frac{L_m}{L_r}\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}\omega_d L_\sigma \underline{i}_s + \mathrm{j}\omega_d \frac{L_m}{L_r}\underline{\phi}_r$$

In steady-state operation and in the synchronous reference frame, the derivatives are zero and the space vectors are replaced by phasors

$$\bar{V}_s = R_s \cdot \bar{I}_s + \mathrm{j}\omega_d L_\sigma \bar{I}_s + \mathrm{j}\omega_d \frac{L_m}{L_r}\bar{\Phi}_r$$

At high speed, the main contribution is due to the back-EMF

The magnitude of the stator voltage phasor can be approximated as

$$V_s \approx \mathrm{j}\omega_d \frac{L_m}{L_r}\Phi_r$$

Since the voltage magnitude is limited (DC-bus and insulation requirements), at high speed the rotor flux must be decreased

A **Flux Weakening** approach can be formulated based on the **magnitude of the rotor flux**

Similar as in scalar control, with the only difference in controlling the rotor flux instead of the stator flux

ROTOR FLUX ESTIMATION

Estimation of the rotor flux vector

NEED FOR ROTOR FLUX ESTIMATION

The field-oriented control is based on the control of the two components of the stator current vector

- The **d-axis** component of the stator currents can control the **flux**
- The **q-axis** component of the stator currents can control the **torque**

But, **the stator current vector must be oriented correctly with respect to the rotor flux vector**

We normally do not have any measurement available on the rotor side

- The **rotor flux** cannot be directly measured, and it **must be estimated**

Different approaches have been proposed, but the basic operation allow to distinguish two main approaches:

- The rotor flux is estimated indirectly **from the rotor circuit model** – **Indirect FOC**
- The rotor flux is estimated directly **from the stator variables** – **Direct FOC**

INDIRECT ROTOR FLUX ESTIMATION

Indirect because the rotor flux is **estimated from the rotor model**

$$\frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r + \frac{1}{T_r}\underline{\phi}_r = \frac{L_m}{T_r}\underline{i}_s$$

(valid for any arbitrary reference frame)

Separated into components leads to:

$$\begin{cases} \dfrac{\mathrm{d}\phi_{r,d}}{\mathrm{d}t} + \dfrac{1}{T_r}\phi_{r,d} - (\omega_d - \omega_e)\phi_{r,q} = \dfrac{L_m}{T_r} i_{s,d} \\ \dfrac{\mathrm{d}\phi_{r,q}}{\mathrm{d}t} + \dfrac{1}{T_r}\phi_{r,q} + (\omega_d - \omega_e)\phi_{r,d} = \dfrac{L_m}{T_r} i_{s,q} \end{cases}$$

The goal is to **align the reference frame to the rotor flux**

If this is achieved, then **all the rotor flux will be positioned on the d-axis, and the q-axis component should be zero**

INDIRECT ROTOR FLUX ESTIMATION

Consider the equation for the q-axis component, and assume that the magnitude of the flux is known

$$\frac{\mathrm{d}\phi_{r,q}}{\mathrm{d}t} + \frac{1}{T_r}\,\phi_{r,q} + (\omega_d - \omega_e)\phi_{r,d} = \frac{L_m}{T_r}\,i_{s,q}$$

We can decide to set the angular frequency of our moving reference frame as:

$$\omega_d = \omega_e + \frac{L_m}{T_r}\,\frac{i_{s,q}}{\phi_{r,d}}$$

In this case, **the presence of the q-axis component of the current is neutralized**, and the equation becomes:

$$\frac{\mathrm{d}\phi_{r,q}}{\mathrm{d}t} + \frac{1}{T_r}\,\phi_{r,q} = 0$$

This means that **the q-axis component of the rotor flux will decrease to zero**
with a first-order dynamics defined by the rotor time constant T_r

In other words, **our moving reference frame will align with the rotor flux position**

INDIRECT ROTOR FLUX ESTIMATION

We can decide to set the angular frequency of our moving reference frame as:

$$\omega_d = \omega_e + \frac{L_m}{T_r}\frac{i_{s,q}}{\phi_{r,d}}$$

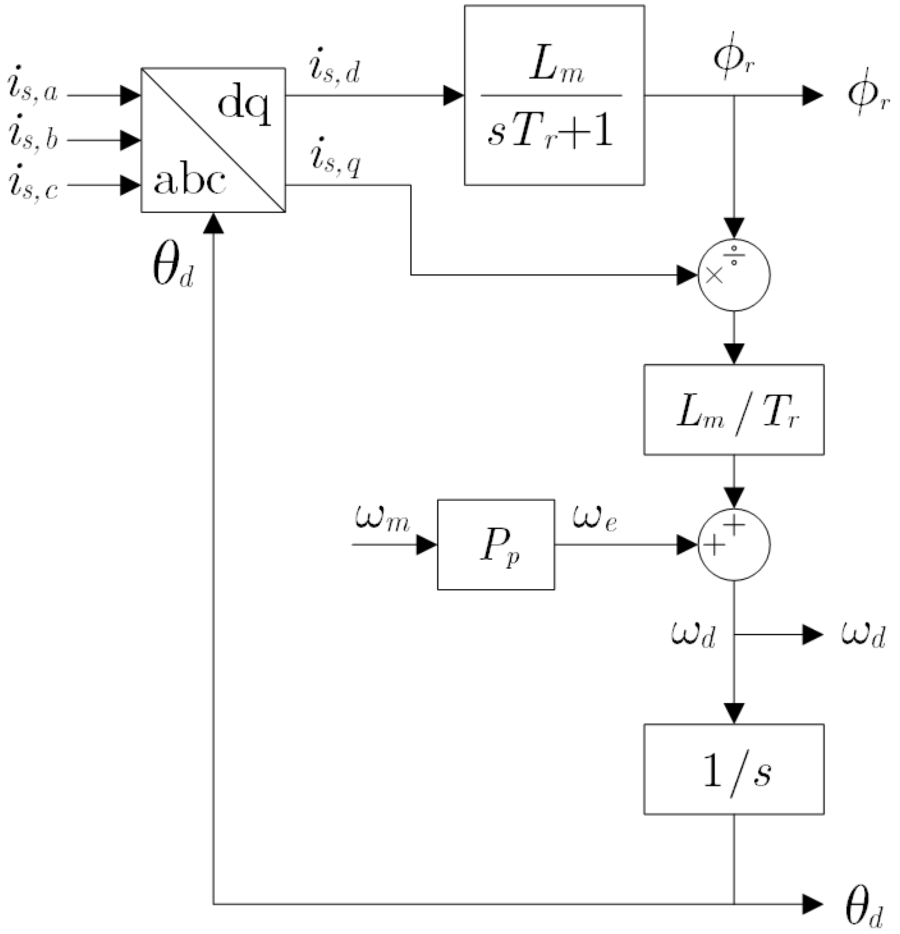
Once the q-axis component of the flux is neutralized, the equation of the d-axis component becomes

$$\frac{\mathrm{d}\phi_{r,d}}{\mathrm{d}t} + \frac{1}{T_r}\phi_{r,d} = \frac{L_m}{T_r} i_{s,d}$$

This means that the d-axis component of the rotor flux evolves following the dynamics of a first order system defined by the rotor time constant T_r

In the Laplace domain, the relationship is represented by a low-pass filter

$$\phi_{r,d}(s) = \frac{L_m}{1 + sT_r}\, i_{s,d}(s)$$



DIRECT ROTOR FLUX ESTIMATION

Direct because the rotor flux is **estimated from the stator voltages and currents**

The calculations are done in the stationary reference frame, considering the stator voltage equation

$$\underline{v}_s = R_s \cdot \underline{i}_s + \frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t}$$

The **stator flux** is determined through integration

$$\underline{\phi}_s = \int (\underline{v}_s - R_s \cdot \underline{i}_s)\mathrm{d}t$$

The voltage vector can be obtained from the reference voltages given at the PWM modulator
(neglecting the voltage drops on the inverter and the delays, they are the average voltages applied to the machine)

With this operation, we know two (vector) variables of the system: stator currents and stator fluxes

Then, **the rotor flux can be expressed as a function of the stator currents and of the stator fluxes**

DIRECT ROTOR FLUX ESTIMATION

The rotor flux can be expressed as a function of the stator currents and of the stator fluxes

The calculations are done in the stationary reference frame, considering the stator voltage equation

$$\underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r$$

$$\underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s$$

From the first equation, we can express the rotor currents as function of the stator currents and stator variables

$$\underline{i}_r = \frac{1}{L_m}\,\underline{\phi}_s - \frac{L_s}{L_m}\,\underline{i}_s$$

This expression can be replaced in the second equation, and the rotor flux is expressed as:

$$\underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s = \frac{L_r}{L_m}\underline{\phi}_s - \frac{L_s\,L_r}{L_m}\underline{i}_s + L_m \cdot \underline{i}_s = \frac{L_r}{L_m}\underline{\phi}_s - \frac{L_r}{L_m}\left(L_s - \frac{L_m^2}{L_r}\right)\underline{i}_s$$

$$= \frac{L_r}{L_m}\underline{\phi}_s - \frac{L_r}{L_m}\sigma\,L_s\,\underline{i}_s = \frac{L_r}{L_m}\left(\underline{\phi}_s - L_\sigma\,\underline{i}_s\right)$$

DIRECT ROTOR FLUX ESTIMATION

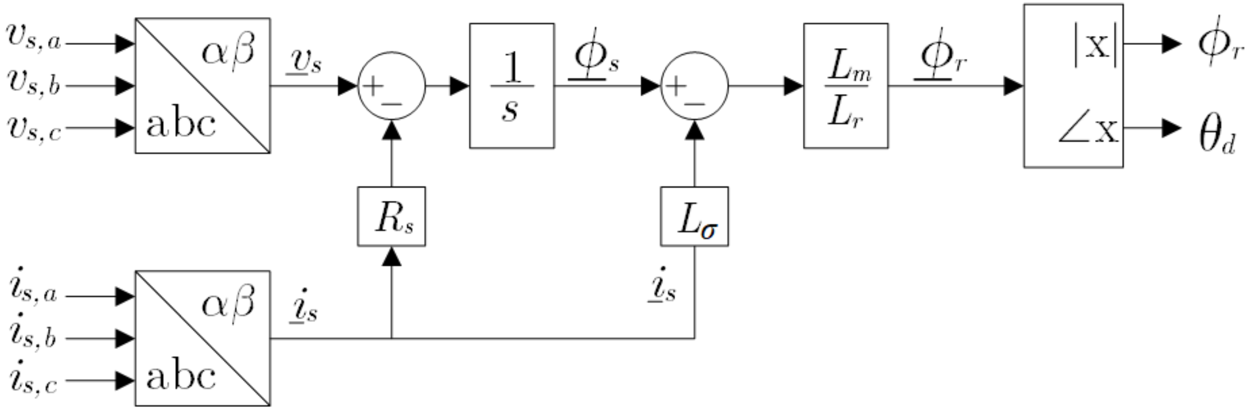
The rotor flux can be expressed as a function of the stator currents and of the stator fluxes

$$\underline{\phi}_r = \frac{L_r}{L_m}\left(\underline{\phi}_s - L_\sigma\ \underline{i}_s\right)$$

This relationship has been obtained in the stationary reference frame

Once expressed in components, it results

$$\begin{cases} \phi_{r,\alpha} = \dfrac{L_r}{L_m}\left(\phi_{s,\alpha} - L_\sigma\ i_{s,\alpha}\right) \\ \phi_{r,\beta} = \dfrac{L_r}{L_m}\left(\phi_{s,\beta} - L_\sigma\ i_{s,\beta}\right) \end{cases}$$



Then, the magnitude and the position of the rotor flux can be computed as:

$$\phi_r = |\underline{\phi}_r| = \sqrt{\phi_{r,\alpha}^2 + \phi_{r,\beta}^2}$$

$$\theta_d = \angle\underline{\phi}_r = \mathrm{atan2}(\phi_{r,\beta}, \phi_{r,\alpha}) = \mathrm{atan}\left(\frac{\phi_{r,\beta}}{\phi_{r,\alpha}}\right)(+\pi)$$

SUMMARY

Vector (Field Oriented) Control

MODEL IN THE REFERENCE FRAME OF THE ROTOR FLUX

The mathematical model of the machine is expressed in terms of stator currents and rotor fluxes

The model is expressed in a rotating reference frame synchronous with the rotor flux

Control of flux and torque are decoupled:

- The **d-axis** component of the stator currents can control the **flux**
- The **q-axis** component of the stator currents can control the **torque**

Stator Current Dynamics (for Current Control)

$$\begin{cases} v_{s,d} = R_{tot} \cdot i_{s,d} + L_\sigma \dfrac{\mathrm{d}i_{s,d}}{\mathrm{d}t} - \omega_d L_\sigma i_{s,q} - \dfrac{R_r\, L_m}{L_r^2}\phi_r \\ v_{s,q} = R_{tot} \cdot i_{s,q} + L_\sigma \dfrac{\mathrm{d}i_{s,q}}{\mathrm{d}t} + \omega_d L_\sigma i_{s,d} + \omega_e \dfrac{L_m}{L_r}\phi_r \end{cases}$$

Rotor flux equations (for Flux and Torque Control)

$$\begin{cases} \dfrac{\mathrm{d}\phi_r}{\mathrm{d}t} + \dfrac{1}{T_r}\,\phi_r = \dfrac{L_m}{T_r}\, i_{s,d} \\ T_{em} = \dfrac{3}{2} P_p \dfrac{L_m}{L_r}\,\phi_r\, i_{s,q} \end{cases}$$

Equivalent Resistance

$$R_{tot} = R_s + R_r \frac{L_m^2}{L_r^2}$$

Equivalent Inductance

$$L_\sigma = \frac{L_s\, L_r - L_m^2}{L_r}$$

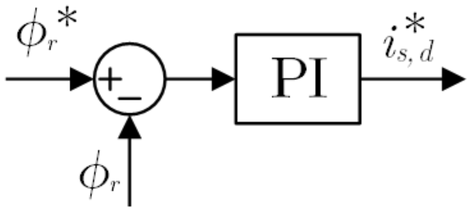
Equivalent back-EMF vector

$$\underline{e}_{eq} = e_d + \mathrm{j}\, e_q = \left(\mathrm{j}\omega_e - \frac{R_r}{L_r}\right) \frac{L_m}{L_r}\, \underline{\phi}_r$$

TORQUE AND FLUX CONTROL

The mathematical model of the machine is expressed in terms of stator currents and rotor fluxes

The model is expressed in a rotating reference frame synchronous with the rotor flux



Control of flux and torque are decoupled:

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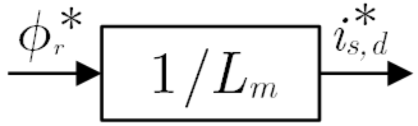
- The **q-axis** component of the stator currents can control the **torque**

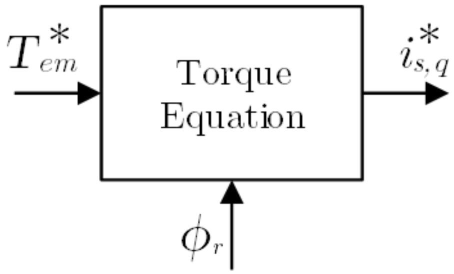
Rotor flux equations (for Flux and Torque Control)

$$\begin{cases} \dfrac{d\phi_r}{dt} + \dfrac{1}{T_r}\phi_r = \dfrac{L_m}{T_r} i_{s,d} \\ T_{em} = \dfrac{3}{2} P_p \dfrac{L_m}{L_r} \phi_r \, i_{s,q} \end{cases}$$



Reference stator currents

$$\begin{cases} i_{s,d}^* = \phi_r^*/L_m \qquad \text{(in steady-state)} \\ i_{s,q}^* = \dfrac{2}{3} \dfrac{L_r}{L_m} \dfrac{1}{P_p} \dfrac{1}{\phi_r} T_{em}^* \end{cases}$$



STATOR CURRENTS CONTROL

The mathematical model of the machine is expressed in terms of stator currents and rotor fluxes

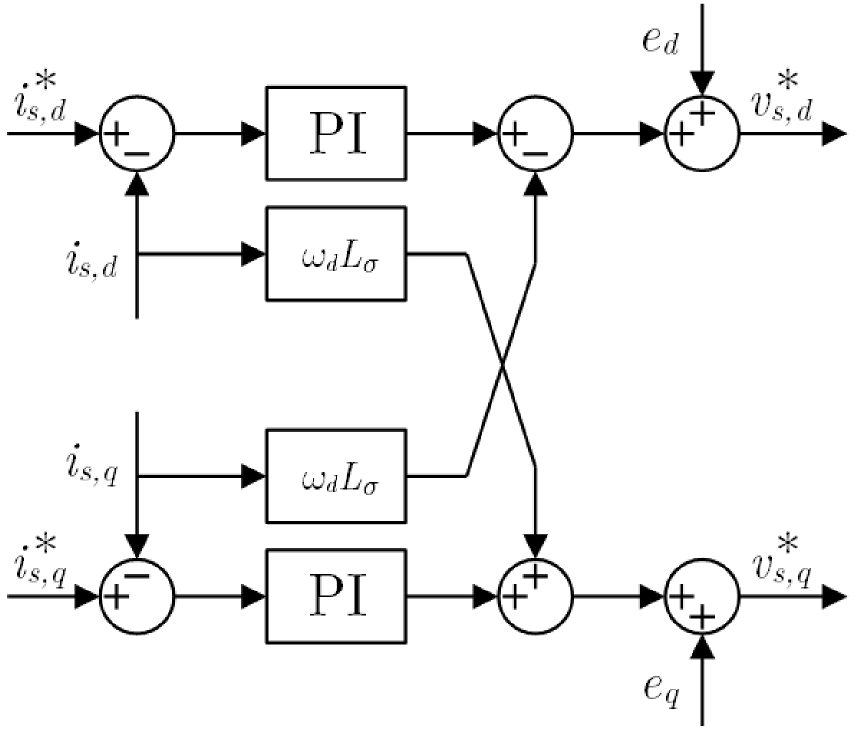
The model is expressed in a rotating reference frame synchronous with the rotor flux

Control of flux and torque are decoupled:

- The **d-axis** component of the stator currents can control the **flux**
- The **q-axis** component of the stator currents can control the **torque**

Stator Current Dynamics (for Current Control)

$$\begin{cases} v_{s,d} = R_{tot} \cdot i_{s,d} + L_\sigma \dfrac{\mathrm{d}i_{s,d}}{\mathrm{dt}} - \omega_d L_\sigma i_{s,q} - \dfrac{R_r L_m}{L_r^2}\phi_r \\ v_{s,q} = R_{tot} \cdot i_{s,q} + L_\sigma \dfrac{\mathrm{d}i_{s,q}}{\mathrm{dt}} + \omega_d L_\sigma i_{s,d} + \omega_e \dfrac{L_m}{L_r}\phi_r \end{cases}$$



The stator currents are controlled with:

- **PI Control** (based on the error between reference and measurement)
- **Cross-Decoupling terms** (to neutralize the d-q axes interference)
- **Back-EMFs Feedforward Compensation** (to neutralize the effect of the rotor flux)

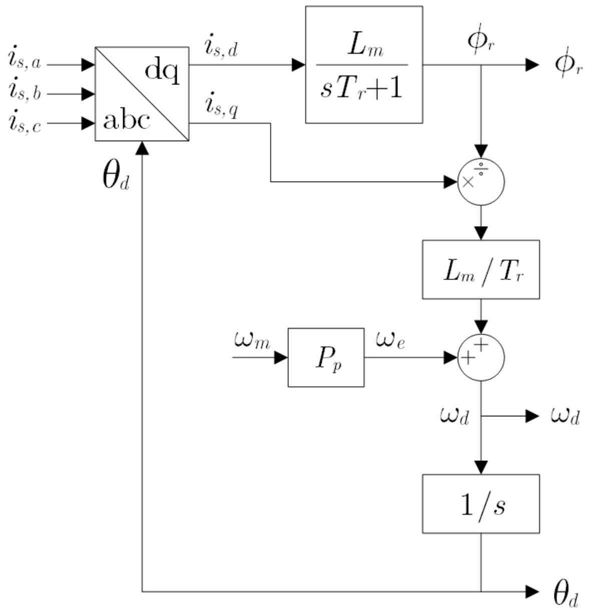
ROTOR FLUX ESTIMATION

The model is expressed in a rotating reference frame synchronous with the rotor flux

The rotor flux position must be estimated properly

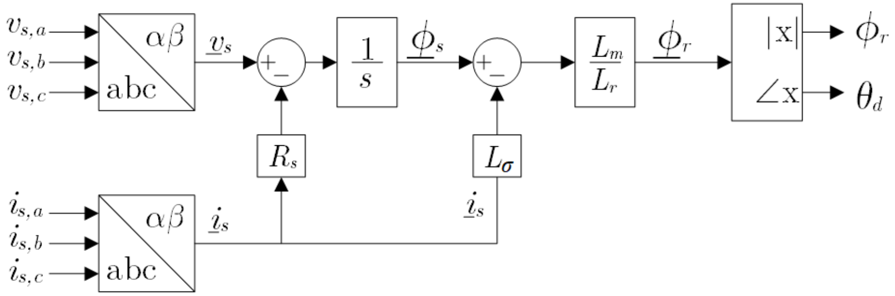
Indirect Rotor Flux Estimation (based on Rotor Equation)

$$\begin{cases} \omega_d = \omega_e + \dfrac{L_m}{T_r}\dfrac{i_{s,q}}{\phi_{r,d}} \\ \dfrac{\mathrm{d}\phi_{r,d}}{\mathrm{d}t} + \dfrac{1}{T_r}\phi_{r,d} = \dfrac{L_m}{T_r} i_{s,d} \end{cases}$$



Direct Rotor Flux Estimation (based on Stator Equation)

$$\begin{cases} \underline{\phi}_s = \displaystyle\int (\underline{v}_s - R_s \cdot \underline{i}_s)\mathrm{d}t \\ \underline{\phi}_r = \dfrac{L_r}{L_m}\left(\underline{\phi}_s - L_\sigma\, \underline{i}_s\right) \end{cases}$$



BLOCK DIAGRAM